Journal of Engineering Physics and Thermophysics, Vol. 70, No. 1, 1997

SPECIFIC FEATURES OF HYDRODYNAMICS AND HEAT TRANSFER IN A FREE-CONVECTIVE NEAR-WALL JET

V. N. Korovkin and A. P. Andrievskii

UDC 536.25

Results of numerical simulation of the development of a laminar free-convective jet along an adiabatic surface are presented. Specific features of the velocity and temperature fields as functions of the Prandtl number are studied. Detailed tables of numerical solutions are given.

Introduction. Interest in the study of free-convective flows above point heat sources has again come into existence recently. This is explained by the fact that many engineering problems associated with convective cooling of electronic circuits and of certain industrial equipment can be successfully solved by the method of superposition [1, 2]. It was found that this approach was as accurate as complex computational schemes and was a valuable supplement to the latter when solving application problems. Since the main idea on which the method of superposition is based is the application of calculation results or dimensionless relations to simple boundary conditions, there arises a need for more detailed studies of free-convective heat transfer above point and linear heat sources.

In what follows we present the results of a comprehensive numerical investigation of hydrodynamics and heat transfer in a flat near-wall free-convective jet based on a model of a laminar boundary layer in a Boussinesque approximation at different Prandtl numbers ($0.1 \le \Pr \le 10$).

Basic Equations. We consider the regime of a stationary laminar fluid flow from a linear heat source imbedded in the leading edge of a vertical semi-infinite adiabatic plate. The physical properties of the fluid (except density) are assumed to be temperature-independent. Then, the basic equations describing a jet vertical flow are presented in the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \quad u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\,(T - T_\infty)\,,$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\frac{\partial^2 T}{\partial y^2}\,. \tag{1}$$

The boundary conditions for this problem are:

$$y = 0:\; v = u = \frac{\partial T}{\partial y} = 0\,, \quad y \to \infty:\; u \to 0\,,\; T \to T_\infty\,. \tag{2}$$

Moreover, the solution of (1)-(2) should also satisfy the condition of conservation of the quantity Q_0

$$Q_0 = C_p \int_0^\infty \rho u\,(T - T_\infty)\,dy = \text{const} \tag{3}$$

in any plane $x = \text{const}$ above the heat source.

We introduce the following transformations:

Polotsk State University, Novopolotsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 70, No. 1, pp. 142-145, January-February, 1997. Original article submitted December 13, 1994.

TABLE 1. Comparison of $f''(0, \mathrm{Pr})$ Values

Pr	[3]	[4]	[5]	Present work
0.001	–	0.20623	–	–
0.01	0.389306	0.38915	–	–
0.1	0.679105	0.67898	–	0.679105
0.3	–	–	–	0.856639
0.4	–	–	–	0.907818
0.5	–	–	–	0.949204
0.7	1.014930	1.01492	1.01572	1.014928
0.72	–	–	–	1.020636
0.73	–	–	–	1.023443
1	1.095027	1.08990	–	1.089880
2	–	–	–	1.255467
5	–	–	–	1.524635
6.7	–	–	–	1.624097
7	–	1.63907	–	1.639570
10	1.394109	1.77096	1.77720	1.771372
100	2.899270	2.89694	2.93779	–
1000	–	4.65865	–	–

TABLE 2. Comparison of $h(0, \mathrm{Pr})$ Values

Pr	[3]	[4]	[5]	Present work
0.001	–	0.05007	–	–
0.01	0.12785	0.12578	–	–
0.1	0.316509	0.31648	–	0.316509
0.3	–	–	–	0.495003
0.4	–	–	–	0.557925
0.5	–	–	–	0.612939
0.7	0.70814	0.70814	–	0.708147
0.72	–	–	0.71688	0.716877
0.73	–	–	–	0.721197
1	0.833970	0.82874	–	0.828748
2	–	–	–	1.142336
5	–	–	–	1.804783
6.7	–	–	2.10364	2.103648
7	–	2.15663	–	2.153006
10	2.606460	2.60609	–	2.606458
100	9.569780	9.55816	–	–
1000	–	37.08893	–	–

TABLE 3. Comparison of Values of f'_{max} and $\eta(f'_{max})$

Pr	[3]		Present work	
	f'_{max}	$\eta(f'_{max})$	f'_{max}	$\eta(f'_{max})$
0.01	0.60829	3.526	–	–
0.1	0.73523	2.320	0.73523	2.319
0.3	–	–	0.75154	1.858
0.4	–	–	0.75028	1.749
0.5	–	–	0.74790	1.667
0.7	0.74229	1.548	0.74227	1.548
0.72	–	–	0.74170	1.538
0.73	–	–	0.74143	1.533
1	0.73407	1.427	0.73405	1.428
2	–	–	0.71271	1.214
5	–	–	0.67458	0.965
6.7	–	–	0.65996	0.893
7	–	–	0.65766	0.883
10	0.63805	0.800	0.63805	0.801
100	0.48414	0.408	–	–

$$\psi = \left(\frac{g\beta Q_0 \nu^2}{\rho C_p}\right)^{1/5} f(\eta)\, x^{3/5}, \quad T - T_\infty = \left(\frac{Q_0}{(g\beta)^{1/4} \rho C_p \sqrt{\nu}}\right)^{4/5} h(\eta)\, x^{-3/5},$$

$$\eta = \left(\frac{g\beta Q_0}{\rho C_p \nu^3}\right)^{1/5} y x^{-2/5}. \tag{4}$$

Then we have instead of (1)

$$f''' + \frac{3}{5} f f'' - \frac{1}{5} f'^2 + h = 0, \quad \frac{1}{\Pr} h'' + \frac{3}{5} f h' + \frac{3}{5} f' h = 0. \tag{5}$$

It follows from (2)-(3) that

$$f(0) = 0, \ f'(0) = 0, \ f'(\infty) = 0; \ h'(0) = 0, \ h(\infty) = 0; \ \int_0^\infty f' h d\eta = 1. \tag{6}$$

Though, using self-similar variables, we succeeded in substantial simplification of the initial problem (1)-(3) (the system of partial differential equations is reduced to a system of ordinary differential equations), analytical relations (5)-(6), both exact and approximate, have not been constructed as yet. Therefore, all calculations of free-convective jet heat transfer are performed numerically within the framework of different schemes [3-6].

Results of Calculations. The nonlinear two-point boundary-value problem (5)-(6) was solved by the standard Runge–Kutta method by reducing it to a Cauchy problem. The missing initial conditions were determined by the shooting technique for a number of values of η_∞ (η_∞ is numerical approximation of a mathematical point $\eta = \infty$) in order to find its independence on η_∞. Moreover, information about the behavior of the characteristics of a jet flow at Pr = 10 was obtained by numerical integration of the modified system of equations

TABLE 4. $f''(0, \mathrm{Pr})$, $h(0, \mathrm{Pr})$, f'_{max} and $\eta(f'_{max})$ as a Functions of Pr

Pr	$f''(0, \mathrm{Pr})$	$h(0, \mathrm{Pr})$	f'_{max}	$\eta(f'_{max})$
0.2924	0.852182	0.489783	0.75155	1.869
0.2926	0.852300	0.489921	0.75155	1.868
0.2928	0.852419	0.490059	0.75155	1.868
0.2930	0.852537	0.490198	0.75155	1.868
0.2932	0.852656	0.490336	0.75155	1.867
0.2934	0.852774	0.490474	0.75155	1.867

$$f_1''' + \frac{1}{\mathrm{Pr}}\left(\frac{3}{5} f_1 f_1'' - \frac{1}{5} f_1'^2\right) + h_1 = 0\,, \quad h_1'' + \frac{3}{5}(f_1 h_1)' = 0\,. \tag{7}$$

Similarly, at Pr = 0.1 an analysis was made by the system

$$\mathrm{Pr}\, f_2''' + \frac{3}{5} f_2 f_2'' - \frac{1}{5} f_2'^2 + h_2 = 0\,, \quad h_2'' + \frac{3}{5}(f_2 h_2)' = 0\,. \tag{8}$$

This was due to the fact that at large (or small) Prandtl numbers the quantities δ_u and δ_T substantially differ from each other. The latter leads to a loss of accuracy of numerical solutions found within the framework of (5)-(6). The use of transformations

$$\begin{gathered} f(\eta) = \mathrm{Pr}^{-3/5} f_1(\eta_1)\,, \quad h(\eta) = \mathrm{Pr}^{3/5} h_1(\eta_1)\,, \quad \eta_1 = \mathrm{Pr}^{2/5}\eta\,; \\ f(\eta) = \mathrm{Pr}^{-2/5} f_2(\eta_2)\,, \quad h(\eta) = \mathrm{Pr}^{2/5} h_2(\eta_2)\,, \quad \eta_2 = \mathrm{Pr}^{3/5}\eta \end{gathered} \tag{9}$$

eliminates this drawback. Then, the numerical algorithm for obtaining the unknown functions was tested on the self-similar problem of a free-convective flow above a linear heat source that has both exact analytical solutions and extensive numerical data [7].

Tables 1-3 give information about the basic hydrodynamic

$$\frac{\tau_w x^2}{\rho \nu^2} = f''(0)\,\mathrm{Gr}_x^{3/5}\,, \quad \mathrm{Gr}_x = \frac{g\beta Q_0 x^3}{\rho C_p \nu^3}$$

and thermal

$$\frac{(T_w - T_\infty)\,\mu C_p}{Q_0} = h(0)\,\mathrm{Gr}_x^{1/5}$$

characteristics of a jet flow developing along an adiabatic surface. It is seen that velocity profiles

$$\frac{ux}{\nu} = f'(\eta)\,\mathrm{Gr}_x^{2/5}\,,$$

similar to a semibounded forced jet, possess a maximum at some distance from the plate. But in a free-convective jet u depends on ΔT. "Competition" between the velocity and temperature fields leads to a nonmonotonic dependence of f'_{max} on Pr: calculation data show that this quantity increases up to a certain Prandtl number and then decreases. Therefore, additional investigations were conducted to determine the threshold value Pr_*. It was found that it lies within the limit $0.2924 < \mathrm{Pr}_* < 0.2934$ (Table 4). A more accurate calculation of Pr_* is difficult, since the global extremum of the function $f'(\eta, \mathrm{Pr})$ turns to be rather mildly sloping. As for the temperature distribution in the flow cross-sections, it has an ordinary form with a maximum at $\eta = 0$ and an asymptotic decrease for $\eta \to \infty$. As Pr grows, profiles $h(\eta)$ become more steep and approach a rectilinear form.

In conclusion, the authors hope that the results presented in this paper will allow one to more deeply comprehend the laws governing stationary free-convective heat transfer in jet flows and also offer new information for engineering calculations.

NOTATION

u, v, longitudinal and transverse velocity components; x, y, longitudinal and transverse coordinates; T, temperature; T_w, T_∞, temperatures of wall and surrounding; ν, kinematic viscosity; Pr, Prandtl number; ρ, density; C_p, heat capacity at constant pressure; $\Delta T = T - T_\infty$, excess temperature; β, coefficient of volumetric heat expansion; δ_u, δ_T, thickness of dynamic and thermal boundary layers; Gr_x, local Grashof number.

REFERENCES

1. Wan, Sovr. Mashinostr. [in Russian], Ser. A, No. 8, 67-72 (1990).
2. Park and Tien, Sovr. Mashinostr. [in Russian], Ser. A. No. 4, 84-91 (1991).
3. J. A. Liburdy and G. M. Faeth, Lett. Heat Mass Transfer, Vol. 2, 407-418 (1975).
4. H.-T. Lin and J.-J. Chen, Int. J. Heat Mass Transfer, 30, No. 8, 1721-1726 (1987).
5. N. Afsal, Int. J. Heat Mass Transfer, 23, No. 4, 505-513 (1980).
6. Y. Jaluria and B. Gebhart, Int. J. Heat Mass Transfer, 20, No. 2, 153-157 (1977).
7. O. G. Martynenko, V. N. Korovkin, and Yu. A. Sokovishin, Theory of Buoyancy Jets and Wakes [in Russian], Minsk (1991).